# LANDMARKS OF SCIENCE IN EARLY AND CLASSICAL INDIA

**Michel Danino**

***Email:*** **micheldanino@gmail.com**

**Based on lectures given at Amrita Vishwa Vidyapeetham, Coimbatore, January 2010, and IIT-Kanpur, February 2010**

# Note

**When we study the history of science in India, we come across a variety of perspectives.**

**At one end of the spectrum, some scholars find no science in ancient India. To them, her contributions are limited, at best, to the decimal place-value system of numeral notation; otherwise, what passes off as Indian science was actually borrowed from the Babylonians, the Greeks, the Chinese and the Arabs.**

**At the other end, overenthusiastic exponents have tried to portray early Indian scientists as creators of virtually all science, including gravitation, the theory of Relativity or quantum physics, and inventors of wondrous machines such as *vimânas* or nuclear weapons.**

**Both stands are untenable. There are many high achievements in the history of science in India, but their development was, as elsewhere, a gradual process, rich in brilliant insights but also, at times, in errors or approximations.**

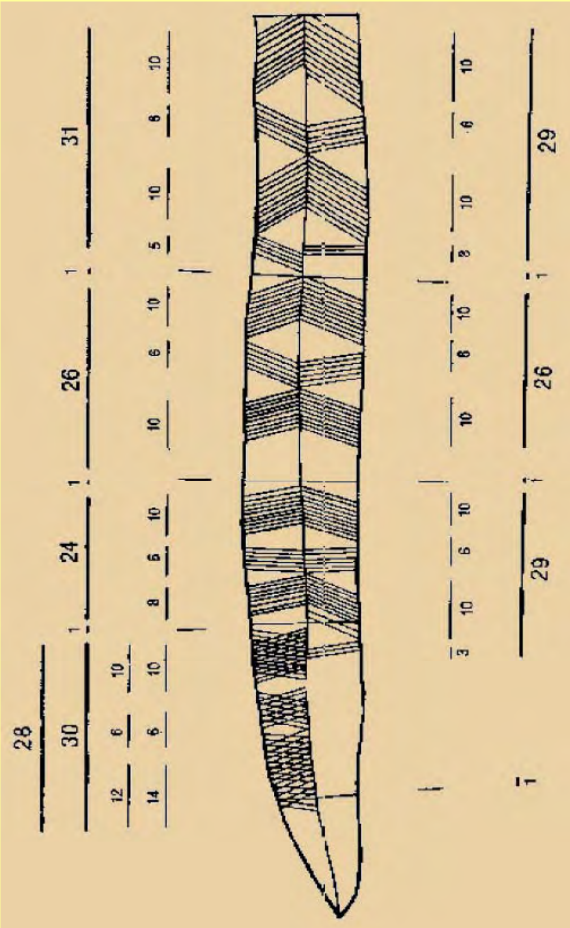
**This presentation offers a overview of the evolution of science in early and classical India; it tries to strike a middle path and to present highlights as objectively as possible. Targeting students, it avoids technical developments and claims no completeness.**

**Comments are welcome.**

# I. Pre-Siddhantic* Developments

* *Siddhânta* = "principle", "conclusion", but in the context of scientific literature, a treatise. This era is inaugurated by Aryabhâta at the end of the 5th century CE.

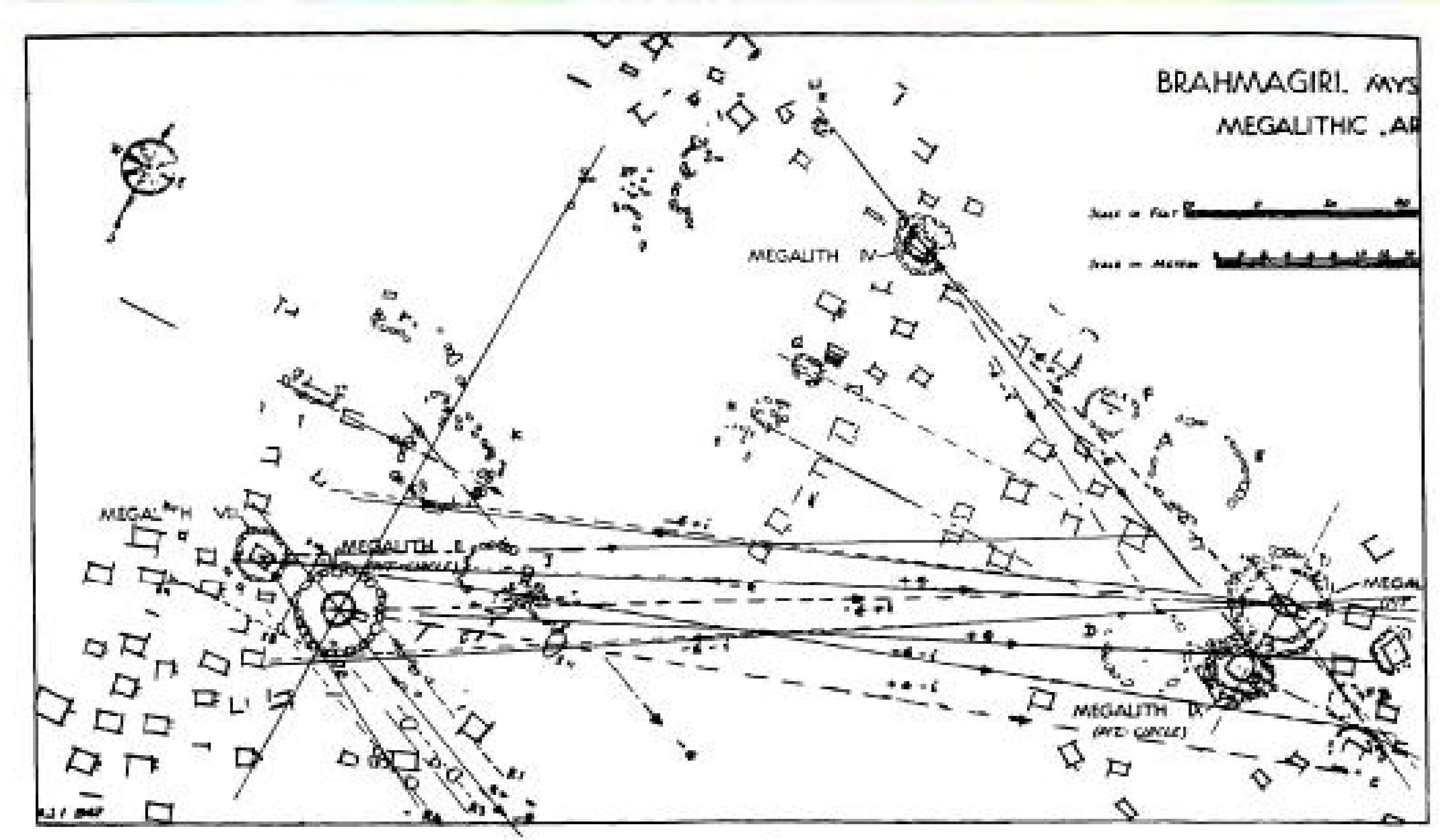
# 1. Neolithic & megalithic antecedents



Calendrical sticks noting the phases of the moon (Andamans, c.10000 BCE)

Burzahom, Kashmir (Neolithic, 3000–1500 BCE). Perhaps a supernova 7,000 years ago. (H. Joglekar, Mayank Vahia & others)

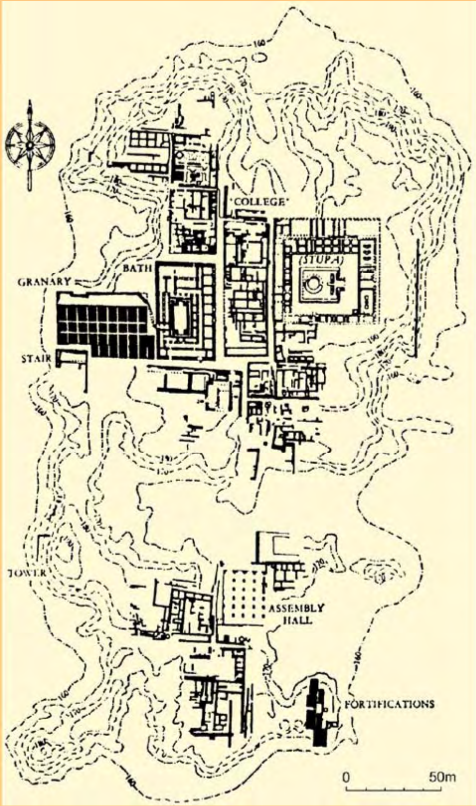
Bomai, Kashmir. The concentric circles may indicate a comet impact. (Mayank Vahia & others)



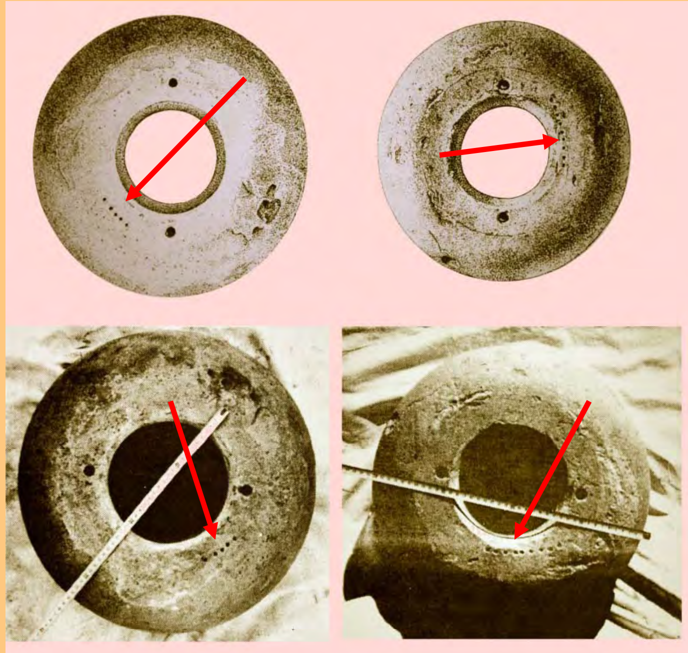

**In every ancient culture, early man regarded the universe as something vast, mysterious and sacred, and tried to relate to it in every possible way: through religion, through customs, and later through science. In megalithic monuments, for instance (above, stone circles at Brahmagiri near Mysore, studied by N.K. Rao), we find alignments with the sunrise at the solar solstice. Such studies of early monuments have long been conducted in the West but are only beginning in India.**

# 2. Indus-Sarasvati Civilization

**First steps of technology and science in the protohistoric era***




**The east-west alignment of the main streets of Mohenjo-daro's "citadel" (or acropolis, *left*) was based on the Pleiades star cluster (*Krittika*), which rose due east at the time; it no longer does because of the precession of the equinoxes. (German archaeologist Holger Wanzke, "Axis systems and orientation at Mohenjo-daro", 1987)**



**The mystery of Mohenjo-daro's "ring stones" *(above right)*: the small drilled holes (see red arrows), showed the stones were used to track the path of the sun through the year, as seen from Mohenjo-daro. Such evidences demonstrate the first steps in observational astronomy. There are other hints, such as possible astronomical symbolism on a few seals. (Finnish scholar Erkka Maulan, "The Calendar Stones from Mohenjo-daro", 1984)**

*** For technology, please see separate pdf file on this civilization.**

# A rudimentary decimal system

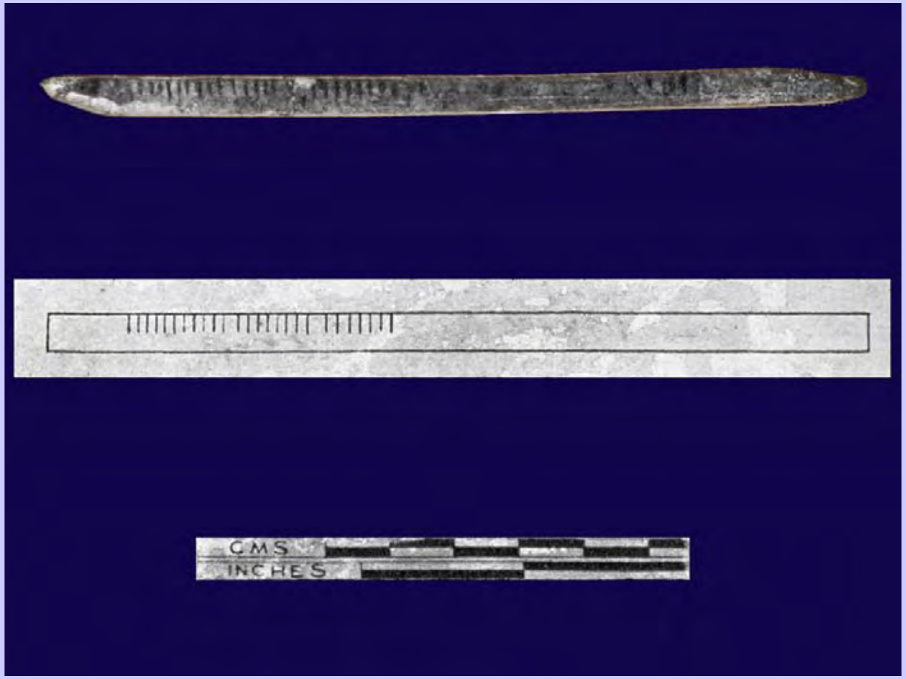
The standardized Harappan system of weights followed a dual binary-decimal progression:

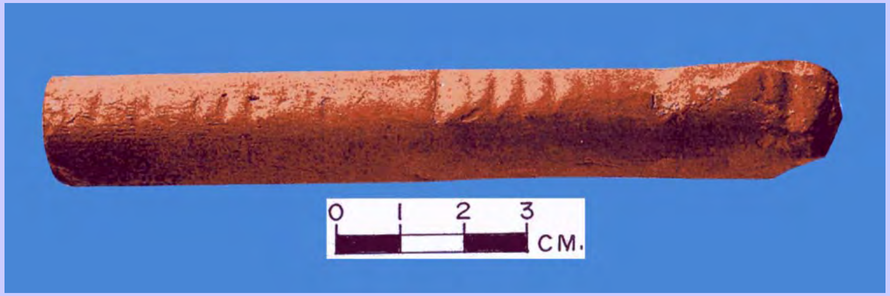
- 1 (= 0.86 g), 2, 4, 8, 16, 32, 64;
- then, instead of continuing with the geometric progression:
- 160, 200, 320, 640, 1,600, 3,200, 6,400, 8,000, 12,800;
- therefore tens, hundreds and thousands of previous units.

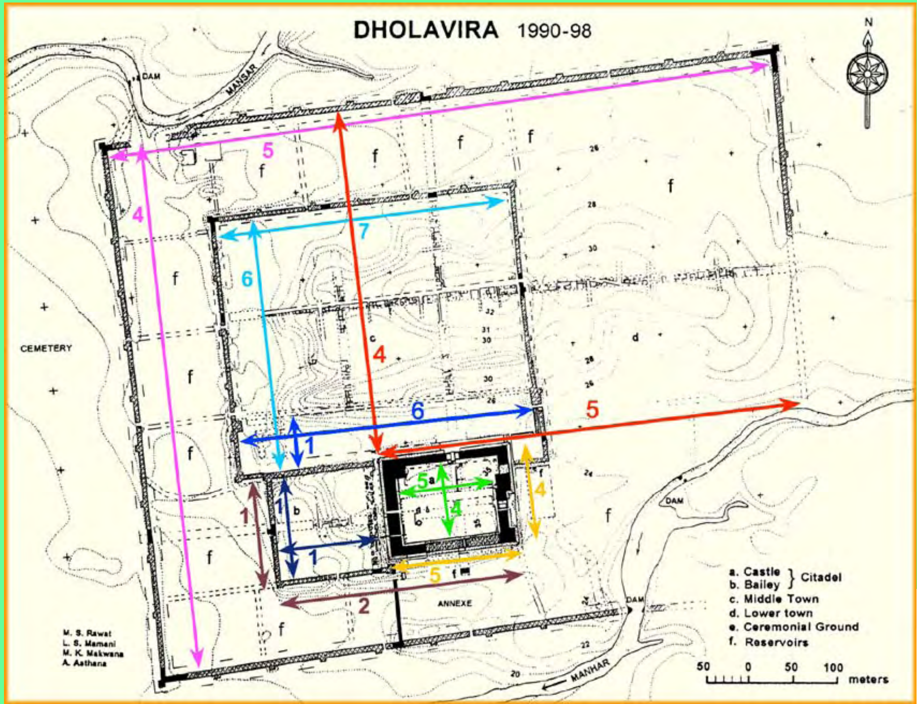
Note: This does not mean that the Harappans noted their numbers in a decimal manner (that is virtually impossible, as this development comes much later). Several other ancient civilizations also used multiples of 10 without a decimal system of numeral notations.

# Linear units related to those of historical times

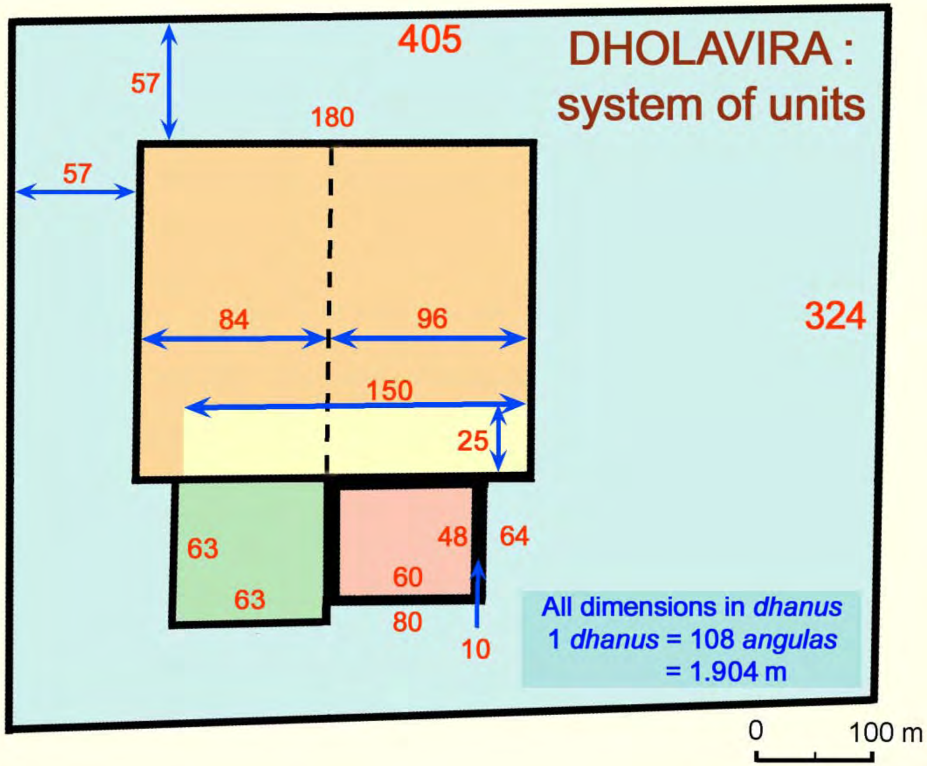
- Lothal's measuring scale (*bottom left,* 27 graduations spanning 46 mm): 1 unit = 1.77 mm.
- V. Mainkar in 1984: 10 Lothal units come close to the *Arthashâstra*'s *angula* or digit (1.778 cm in his estimate).
- Kalibangan's terracotta scale: a unit of 1.75 cm (analyzed by Prof. R. Balasubramaniam, IIT-Kanpur).
- In contemporary Egypt and Mesopotamia (later in China, Greece, Japan, or the Roman Empire), the traditional digit fluctuated between 1.6 and 1.9 cm .



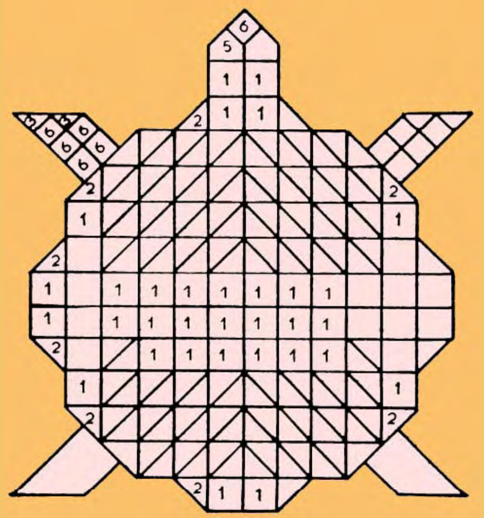

- **Dholavira's successive enclosures follow strict ratios *(left)*, evidence of specific concepts of auspiciousness, sacred proportions, etc.**
- **It is possible to calculate the unit of length used in planning Dholavira: 1.9 m (108 times 1.76 cm, the Harappan *angula*). (Research by Michel Danino.)**



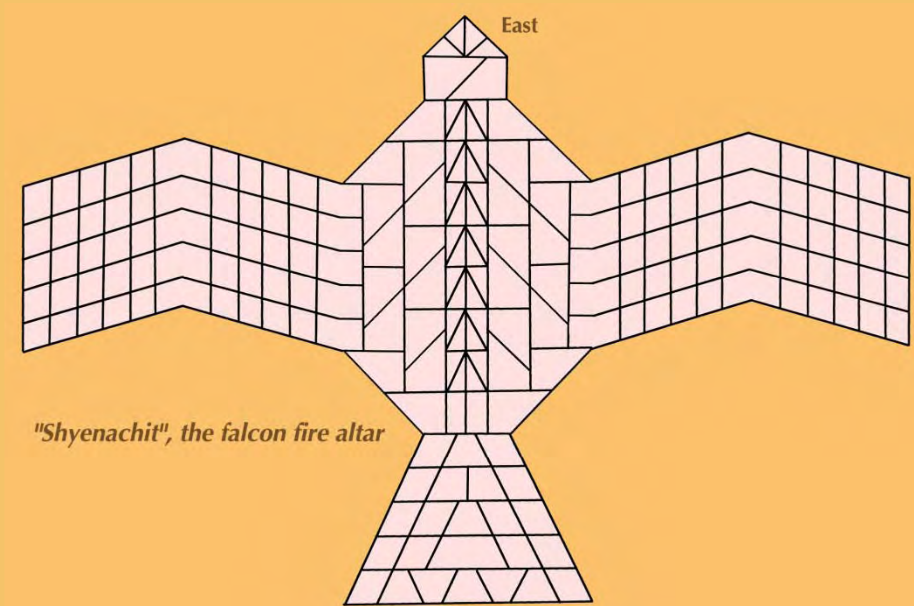
# 3. Historical Era: pre-Siddhantic* mathematics

*** *Siddhânta* = principle, conclusion, but in the context of scientific literature, a treatise. This era is inaugurated by Aryabhâta at the end of the 5th century CE.**

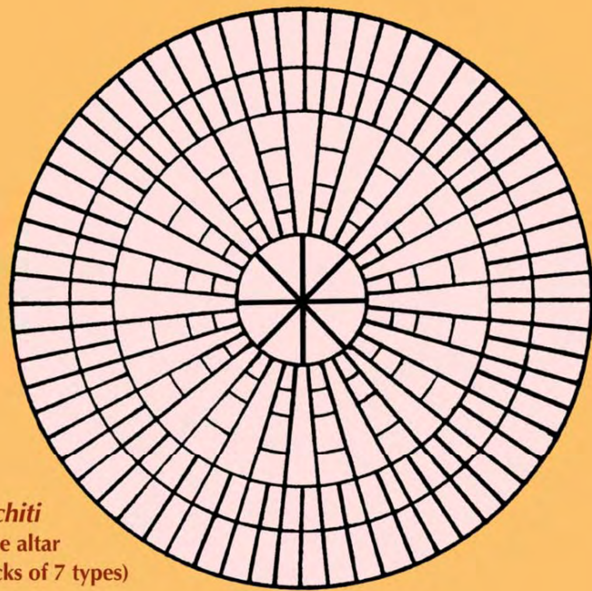
- **Geometry of the *Shulbasûtras* (8th to 6th century BCE: *shulba* = rope): these four ancient texts deal with fire altars of various shapes. The altar, with 5 layers of 200 bricks each, symbolizes the universe, from the earth to the highest world.**
- **The altars were constructed with bricks of specific shapes and area: the total area of the altar must always be carefully respected. It was generally 7½ square *purushas,* the *pususha* being, in the *Shulbasûtras* , the height of a man with his arm raised above his head.**
- **This leads to precise but purely geometrical calculations (algebra did not exist yet).**



*Kurmachit fire altar - first layer*
(with 6 types of bricks totalling 200)

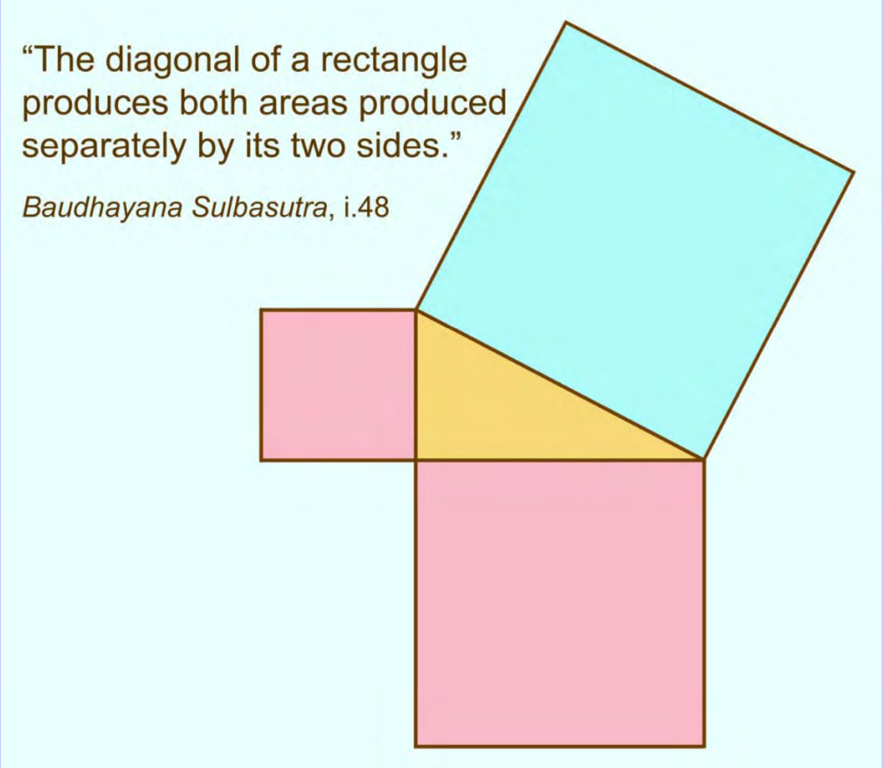



*"Shyenachit", the falcon fire altar*



The *rathachakrachiti*
or chariot-wheel fire altar
(first layer, 200 bricks of 7 types)

- The *Shulbasûtras* give a precise geometric expression of the so-called "Pythagorean theorem."
- Right angles were made by ropes marked to give the triads 3, 4, 5 and 5, 12, 13 ($3^2 + 4^2 = 5^2$, $5^2 + 12^2 = 13^2$).
- We should rename this theorem the "Shulba theorem"!




## Examples of other geometric operations in the *Shulbasûtras*:

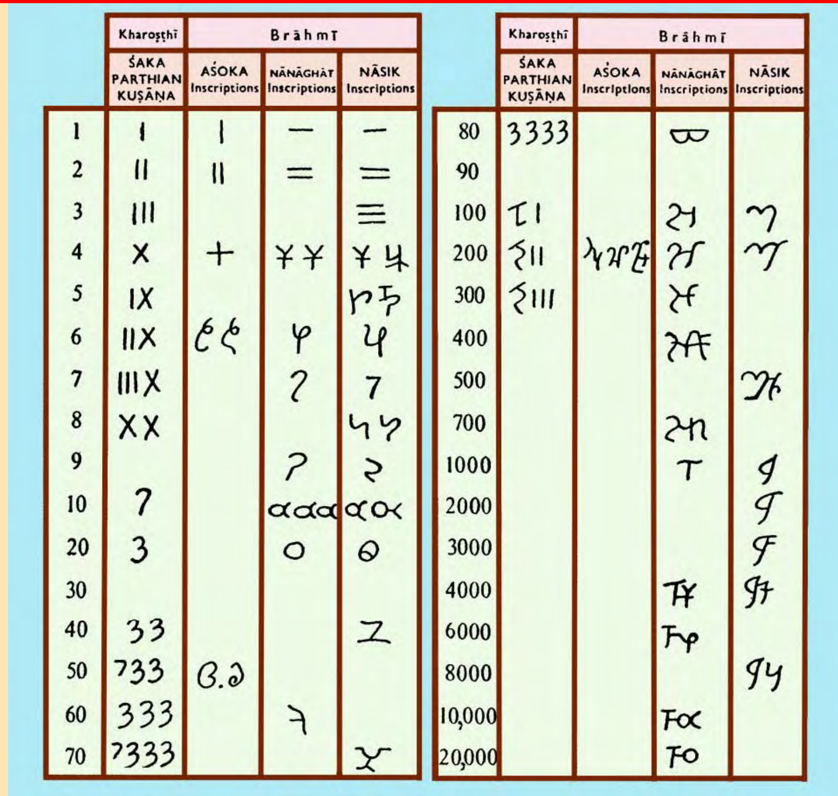
- "Squaring the circle" (and vice-versa): geometrically constructing a square having the same area as a given area.
- Adding or subtracting the areas of two squares (to produce a single square).
- Doubling the area of a square.
- In the last construction, $\sqrt{2}$ works out to 577/408 or 1.414215, correct to the 5th decimal. (Same precision with $\sqrt{3}$.)

| | 1 2 4 8 16 32 | | 1 2 4 8 16 32 | | 1 2 4 8 16 32 | | 1 2 4 8 16 32 |
|---|---|---|---|---|---|---|---|
| 1. | – – – – – – | 17. | – – – – ⏑ – | 33. | – – – – – ⏑ | 49. | – – – – ⏑ ⏑ |
| 2. | ⏑ – – – – – | 18. | ⏑ – – – ⏑ – | 34. | ⏑ – – – – ⏑ | 50. | ⏑ – – – ⏑ ⏑ |
| 3. | – ⏑ – – – – | 19. | – ⏑ – – ⏑ – | 35. | – ⏑ – – – ⏑ | 51. | – ⏑ – – ⏑ ⏑ |
| 4. | ⏑ ⏑ – – – – | 20. | ⏑ ⏑ – – ⏑ – | 36. | ⏑ ⏑ – – – ⏑ | 52. | ⏑ ⏑ – – ⏑ ⏑ |
| 5. | – – ⏑ – – – | 21. | – – ⏑ – ⏑ – | 37. | – – ⏑ – – ⏑ | 53. | – – ⏑ – ⏑ ⏑ |
| 6. | ⏑ – ⏑ – – – | 22. | ⏑ – ⏑ – ⏑ – | 38. | ⏑ – ⏑ – – ⏑ | 54. | ⏑ – ⏑ – ⏑ ⏑ |
| 7. | – ⏑ ⏑ – – – | 23. | – ⏑ ⏑ – ⏑ – | 39. | – ⏑ ⏑ – – ⏑ | 55. | – ⏑ ⏑ – ⏑ ⏑ |
| 8. | ⏑ ⏑ ⏑ – – – | 24. | ⏑ ⏑ ⏑ – ⏑ – | 40. | ⏑ ⏑ ⏑ – – ⏑ | 56. | ⏑ ⏑ ⏑ – ⏑ ⏑ |
| 9. | – – – ⏑ – – | 25. | – – – ⏑ ⏑ – | 41. | – – – ⏑ – ⏑ | 57. | – – – ⏑ ⏑ ⏑ |
| 10. | ⏑ – – ⏑ – – | 26. | ⏑ – – ⏑ ⏑ – | 42. | ⏑ – – ⏑ – ⏑ | 58. | ⏑ – – ⏑ ⏑ ⏑ |
| 11. | – ⏑ – ⏑ – – | 27. | – ⏑ – ⏑ ⏑ – | 43. | – ⏑ – ⏑ – ⏑ | 59. | – ⏑ – ⏑ ⏑ ⏑ |
| 12. | ⏑ ⏑ – ⏑ – – | 28. | ⏑ ⏑ – ⏑ ⏑ – | 44. | ⏑ ⏑ – ⏑ – ⏑ | 60. | ⏑ ⏑ – ⏑ ⏑ ⏑ |
| 13. | – – ⏑ ⏑ – – | 29. | – – ⏑ ⏑ ⏑ – | 45. | – – ⏑ ⏑ – ⏑ | 61. | – – ⏑ ⏑ ⏑ ⏑ |
| 14. | ⏑ – ⏑ ⏑ – – | 30. | ⏑ – ⏑ ⏑ ⏑ – | 46. | ⏑ – ⏑ ⏑ – ⏑ | 62. | ⏑ – ⏑ ⏑ ⏑ ⏑ |
| 15. | – ⏑ ⏑ ⏑ – – | 31. | – ⏑ ⏑ ⏑ ⏑ – | 47. | – ⏑ ⏑ ⏑ – ⏑ | 63. | – ⏑ ⏑ ⏑ ⏑ ⏑ |
| 16. | ⏑ ⏑ ⏑ ⏑ – – | 32. | ⏑ ⏑ ⏑ ⏑ ⏑ – | 48. | ⏑ ⏑ ⏑ ⏑ – ⏑ | 64. | ⏑ ⏑ ⏑ ⏑ ⏑ ⏑ |

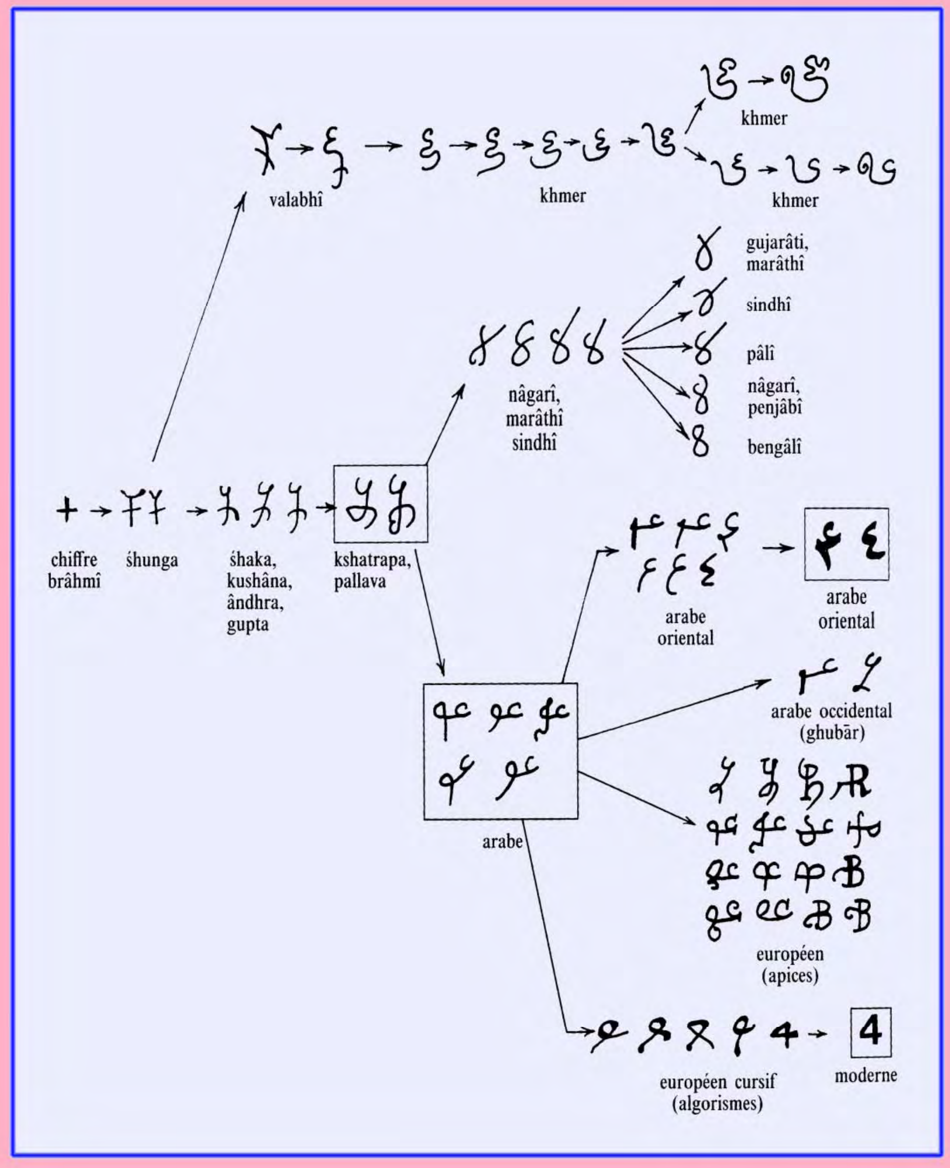
The *prastāra* for six syllables from Pingala's *Chandasastra*

- **Pingala's treatise *Chhandashâstra* or the science of verse meters (a few centuries BCE, Pingala being perhaps the brother of the famous grammarian Panini).**
- **Notation of verse meters for verses with various numbers of syllables (6 in this case).**
- **Syllables are light (*laghu*) or heavy (*guru*). The system of notation, spelling out every possible combination of light and heavy syllables, reflects the concept behind the modern binary notation of numbers used in computers.**

**Earliest inscriptions (first centuries BCE and CE): numerals without decimal place value. See for instance how in the first column, 40 is formed by repeating the symbol for 20 twice. There is also no symbol for 0 and no mathematical concept of zero.**



- It is well established that so-called “Arabic numerals” originated in India. Their evolution has been traced to the Brâhmî script of Mauryan times.
- The Webster dictionary gives the synonym of “Hindu-Arabic numerals.”
- All Indian numerals are also ultimately derived from Brâhmî numerals (except for Tamil, which had a different system using letters).



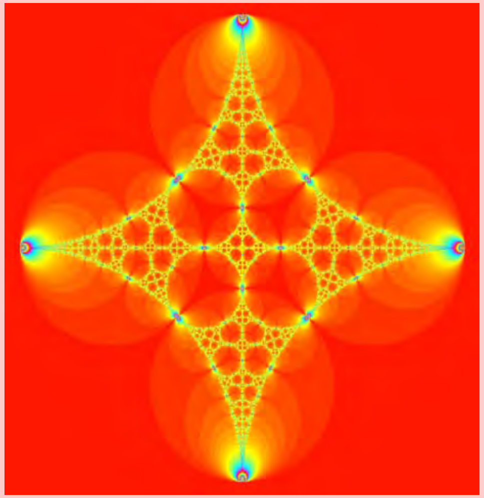
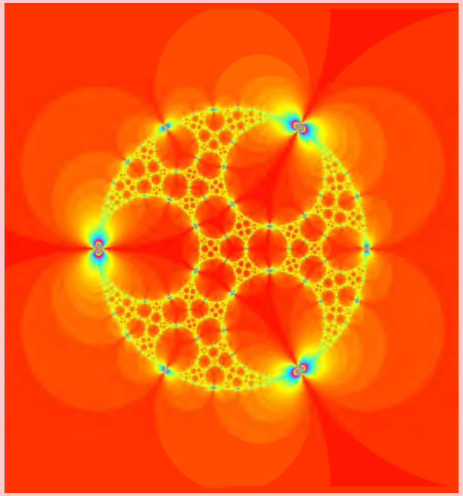

*Evolution of the numeral 4 from Brahmi to the modern (by Georges Ifrah)*

# Numbers 1 to 10 and symbolic connotations

| | |
|---|---|
| 0 - *shunya* | *bindu, kha, purna, vyoman, akasha, ambara, ananta…* |
| 1 - *eka* | *adi, pitamaha, tanu, kshiti, indu, soma, atman, brahman, surya…* |
| 2 - *dvi* | *ashvin, netra, paksha, yama...* |
| 3 - *tri* | *guna, loka, kala, agni, tripura, trishul, rama...* |
| 4 - *chatur* | *dish, yuga, irya, ashrama, veda...* |
| 5 - *pañcha* | *ishu, indriya, bhuta, pandava...* |
| 6 - *shat* | *rasa, anga, shanmukha, darshana...* |
| 7 - *sapta* | *ashva, naga, rishi, sagara, dvipa, buddha, sindhava, matrika...* |
| 8 - *ashta* | *gaja, naga, murti...* |
| 9 - *nava* | *anka, graha, chhidra, ratna...* |
| 10 - *dasha* | *anguli, asha, avatara, dish...* |

**Another example of the interface between science and spirituality in ancient India is found in the many names that numbers were given in scientific texts. Most of the names had philosophical or mythological backgrounds. The word *râma*, for instance, does not refer to Lord Rama but to number 3, because there were 3 Râmas (Parashurâma, Râma and Balarâma).**

**Find out how many names you can explain in the above list.**

## Indra’s Pearls

Mathematical concepts are also found in the background of purely religious texts. The *Avatamsaka Sûtra* (a Buddhist text, a few centuries BCE), for instance, depicts a network of pearls placed in heavens by Indra so that “in each pearl one can see the reflections of all the others, as well as the reflections within the reflections and so on.” Three U.S. mathematicians found that Indra’s pearls follow the arrangement of circles in a “Schottky group” (and published a book entitled *Indra’s Pearls*). Two such structures are shown above.

Of course, the Buddhist writers were unaware of these mathematical structures, but they seem to have had an intuition of the concept behind them.

Modern science’s investigation of such ancient concepts is an interesting exercise.

# 4. Historical era: pre-Siddhantic astronomy

The *Rig-Veda* clearly refers to an eclipse: the sun is struck with darkness by an asura (Svarbhânu). The eclipse was dated to 26 July 3928 BCE by P.C. Sengupta, but interpreting the texts to derive dates is always a delicate exercise.

(Photo: annular solar eclipse of 15 January 2010 seen from Coimbatore)

*"Swift and all beautiful art thou,*
*O Surya, maker of the light,*
*illuming all the radiant realm"*
(Rig-Veda, 1.50.4)

Sayana comments thus on the above hymn :

*tathā ca smaryate yojanānām*
*sahasre dve dve sate dve ca yojane*
*ekena nimisārdhena kramamāna*

"Thus it is remembered :
[O Surya] you who traverse
2,202 *yojana* in half a *nimesa*."

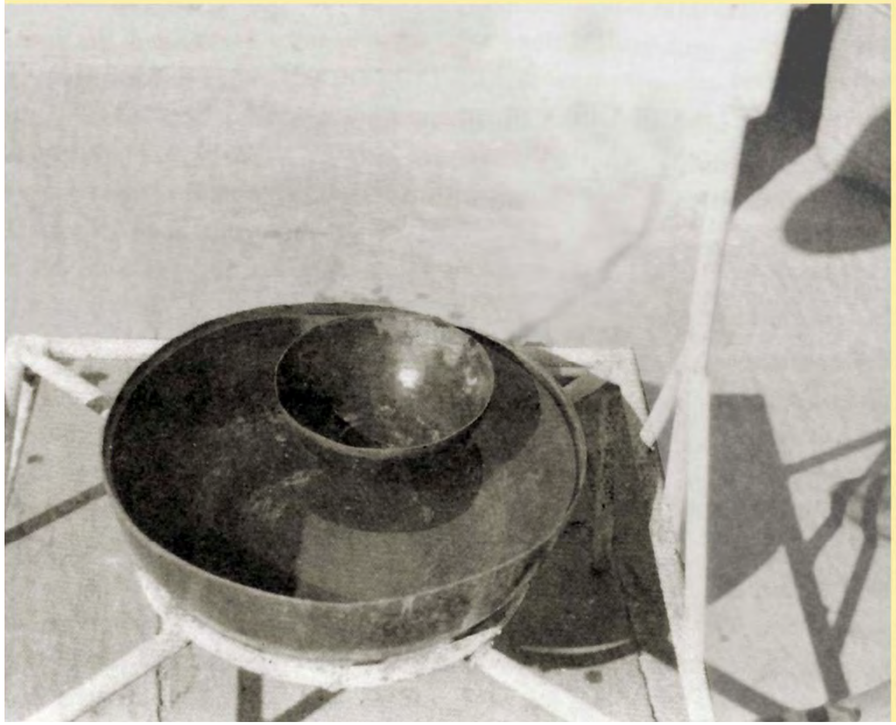
**The above commentary on the Rig-Veda, by Sâyana, was composed in the 15th century and refers to an ancient tradition ("it is remembered"). With a *yojana* of 13.6 km (the *yojana* is sometimes defined as 8000 average human heights of 1.7m) and a *nimesa* of 16/75$^{th}$ of a second, this amounts to 280,755 km/s — just 6% from the speed of light (299,792 km/s). There are several possible explanations:**

- **A coincidence? But if so, why be so precise (2,202 and not 2,000)?**
- **Some lost method of measurement of the speed of light?**
- **Intuition or inspired knowledge?**

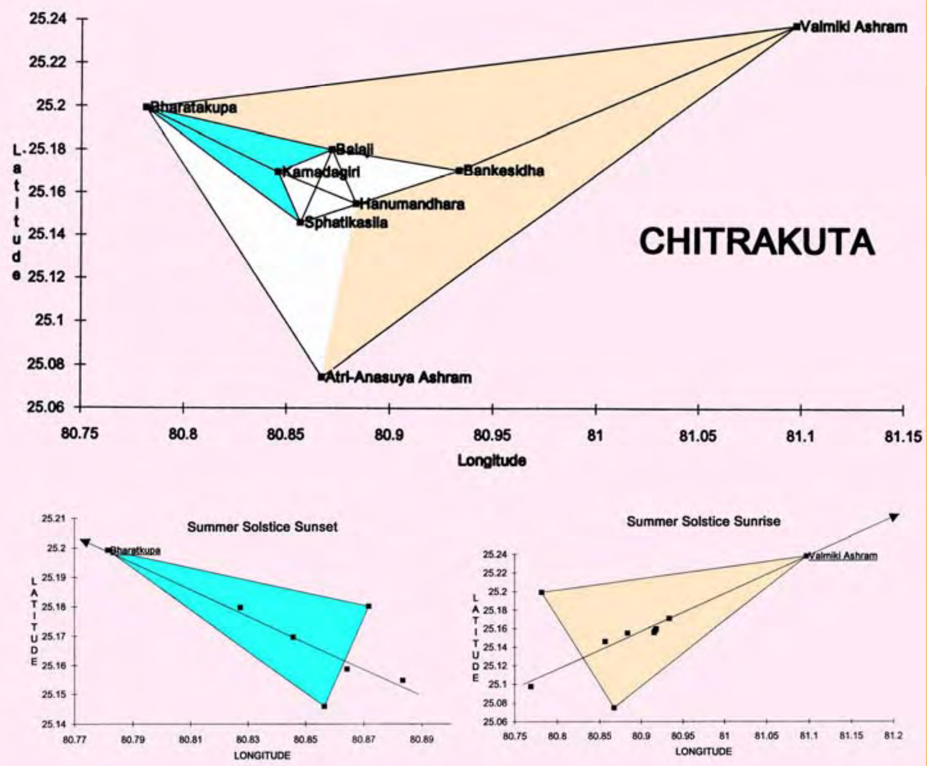
**In any case, the fact should be noted.**

# Other important landmarks of astronomy:

- Lagadha's *Vedanga Jyotisha,* India's earliest scientific text, is dated about 1400 BCE. It spells out seasons, the 17 *nakshatras* or lunar mansions, the concept of a 5-year yuga, units of time and time division, the number of lunar days, months and cycles in a yuga, and rules for a calendar.
- Texts of Jaina astronomy develop a system of two suns, two moons and two sets of 27 *nakshatras*.

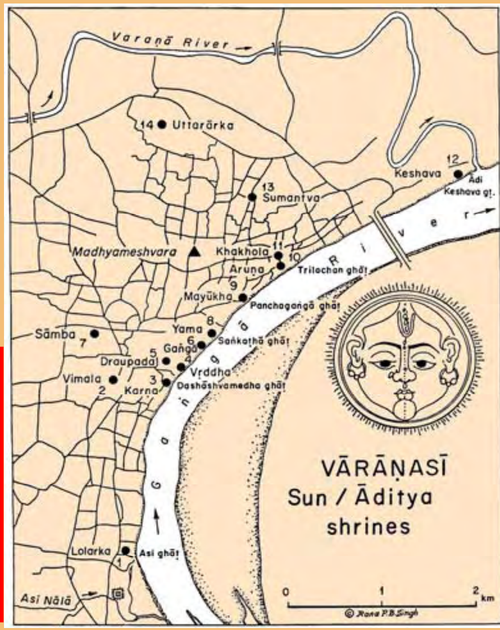


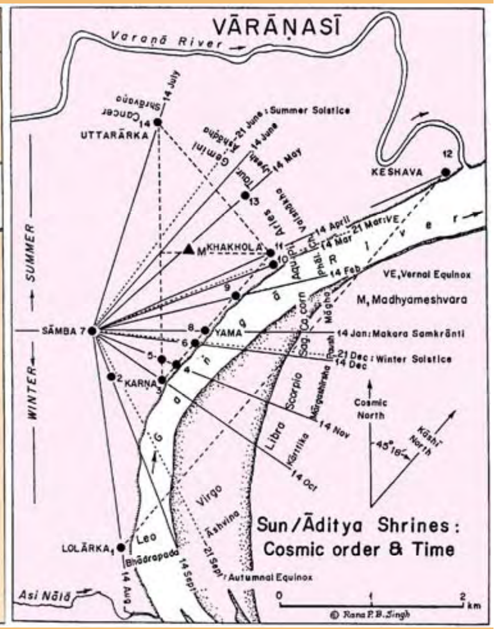
- We find also evidence of a tradition of astronomical observation.
- The *ghati yantra (left)* was a type of water clock: the bowl of bronze, with a small calibrated hole at its bottom, sinks after 24 mn (a unit of time called *ghati*, equal to 1/60th of a day).
- Ancient texts refer to various other devices such as gnomons, sun dials, which have disappeared, but point to a long tradition of observation.




The universe can be symbolized not just by an altar, but in a whole site or even a city. U.S. astrophysicist McKim Malville, with Indian scholars, studied India's sacred geography: at Varanasi, Chitrakut, Vijayanagara and other places, sacred sites (shrines, ashrams etc.) were oriented towards specific points of the sun's path across the year. This represents the same need to connect the microcosm to the macrocosm, and shows the potential of such lines of scientific inquiry.

Chitrakut (associated with Lord Ram, who is often represented by a symbolic as an arrow): once mapped with GPS, ashrams form arrows that point to the sunrise and sunset on the summer solstice.




Varanasi: the 14 Aditya shrines precisely track the path of the sun through the year, month after month, in an exact mapping on the ground of the Hindu calendar.

# India was a pioneer in many technologies.


- Metallurgy (bronze, iron, wootz, zinc...)
- Pottery (ceramic, faience...)
- Pigments (painting, dyeing...)

- Perfumes & cosmetics
- Medicines
- Chemistry and alchemy (involving numerous processes of transformation of matter)

Ancient India’s steel was prized all over the Greek and Roman world for its ability to make sharp and thin swords. There is a Roman account that when Alexander defeated Porus, he asked a tribute not of gold or jewels, but of Indian steel (more than 2 tons of it).

The map *(right)* shows sites where wootz steel was prepared in South India (note Kodumanal near Coimbatore, dated about 500 BCE).


The Delhi Iron Pillar: a thin layer of iron and phosphorus compound made it rustproof, even after more than 1,500 years. (Process identified by R. Balasubramaniam, on the photo.)

# 5. The growth of the Indian scientific mind

## Indian time scale ← *compare* → Judeo-Christian time scale

| | |
|---|---|
| Satya: | 1,728,000 years |
| Tretâ: | 1,296,000 years |
| Dvâpara: | 864,000 years |
| Kali: | 432,000 years |

***Chaturyuga*** **:** **4,320,000 years**

**Duration of a "day of Brahmâ" = one *kalpa* or 1,000 *chaturyuga* = 4.32 billion years.**

***Anno mundi*** **(year of the world's creation):**

- **3761 BC (Judaism)**
- **4004 BC (Christianity)**

**"The Hindu religion is the only one of the world's great faiths dedicated to the idea that the Cosmos itself undergoes an immense, indeed an infinite, number of deaths and rebirths. It is the only religion in which the time scales correspond, no doubt by accident, to those of modern scientific cosmology. Its cycles run from our ordinary day and night to a day and night of Brahma, 8.64 billion years long. Longer than the age of the Earth or the Sun and about half the time since the Big Bang. And there are much longer time scales still."**

**U.S. astronomer Carl Sagan, *Cosmos***

# Intuition of the relativity of time

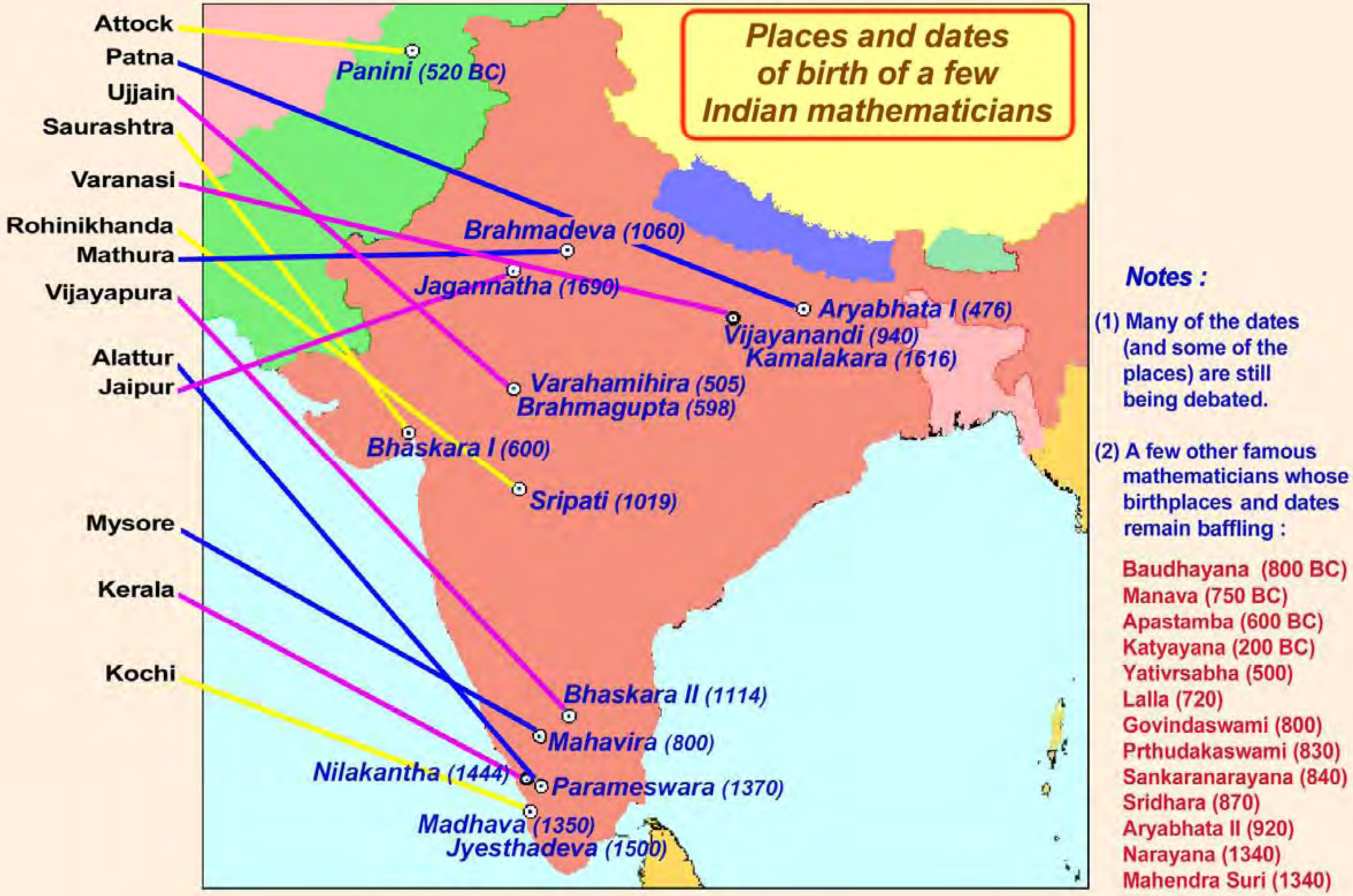
**Taking his own daughter, Revati, Kakudmi went to Lord Brahma in Brahmaloka, and inquired about a husband for her. When Kakudmi arrived there, Lord Brahma was engaged in hearing musical performances by the Gandharvas and had not a moment to talk with him. Therefore Kakudmi waited, and at the end of the performance he saluted Lord Brahma and made his desire known. After hearing his words, Lord Brahma laughed loudly and said to Kakudmi, "O King, all those whom you may have decided within the core of your heart to accept as your son-in-law have passed away in the course of time. Twenty-seven chaturyugas have already passed. Those upon whom you may have decided are now gone, and so are their sons, grandsons and other descendants. You cannot even hear about their names."**

**(*Bhagavata Purana*)**

# Intuition of the concept of evolution

- The notion of *Dashavatar* (10 incarnations of the divine consciousness) contains the seed of the concept of evolution: the first body is a fish, the second an amphibian, the third a mammal, the fourth half-man half-animal, the fifth a short man, etc. (Later stages reflect a spiritual evolution.)
- This implies a conceptualization or intuition of the truth expressed by Darwinian evolution (but note that it would be wrong to claim that ancient Indians had discovered Darwinian evolution).

# II. The Siddhantic Era




# Early Indian scientists

- **This map (adapted from the website of St. Andrews University, Scotland) lists the main figures of early Indian science. (The exact place or epoch of many of them remains uncertain).**
- **Note the shift to the South, especially Karnataka and Kerala, after the 12th century.**

# How much do we know?

In 2002, the great scholar K.V. Sarma published a list of 3,473 science texts compiled from 12,244 science manuscripts found in 400 repositories in Kerala and Tamil Nadu.

Of those 3,473 texts, <u>no more than 7% are available in print</u> (and still less in translation).

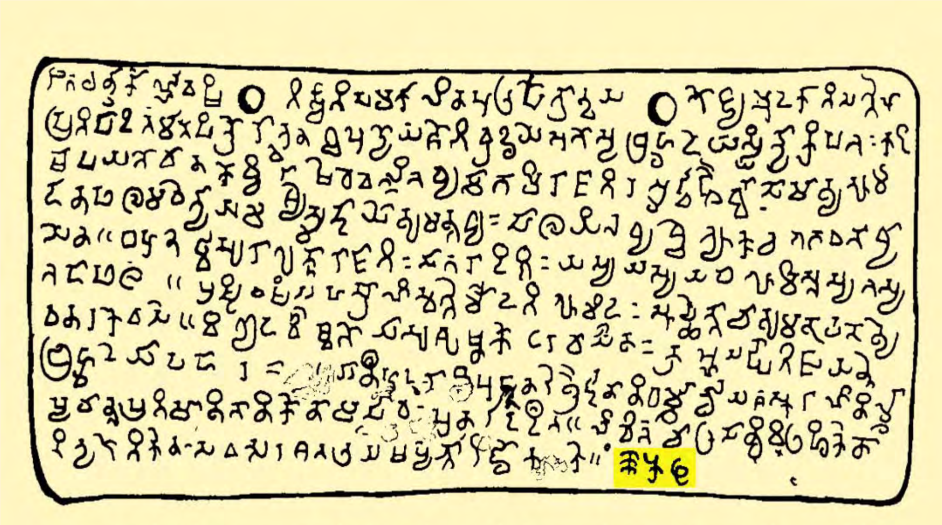
> Historians of science know nothing of the remaining 93% and thus regard this 7% "as the whole and sole of the contribution of India to science. This is a sorry situation from which Indian science has to be rescued and resurrected". (K.V. Sarma)

Unfortunately, in India history of science is not a well developed academic discipline (unlike in the West).

This means our knowledge of past scientific achievements is fragmentary.

# Advances in mathematics

The first known inscription with a decimal place-value notation (Sankheda, Gujarat, dated “346” Chhedi era, or AD 594): for the first time, “3” stands for hundreds, “4” for tens and “6” for units.



Georges Ifrah: ***The Universal History of Numbers,*** **in 3 volumes. Volume 2 is mostly about India’s contributions to mathematics.**

Sessa’s story (from Persian and Arabic sources) of the Brahmin explaining the game of chess to a king and asking for a reward of 1 grain of rice on the first square, two on the second, etc., illustrates the power of the decimal place-value system of numeral notation.


*Viyadambarâkâshashunyayamarâmaveda*

|

*viyat-ambara-âkâsha-shunya-yama-râma-veda*

|

sky-atmosphere-ether-void-Yama-Râma-Veda

|

0-0-0-0-2-3-4

|

432,0000

(= the number of years in Kali Yuga)

A number from Bhaskara: in mathematical texts, numbers were often noted not with numerals, but with names having a mythological meaning related to a particular number.

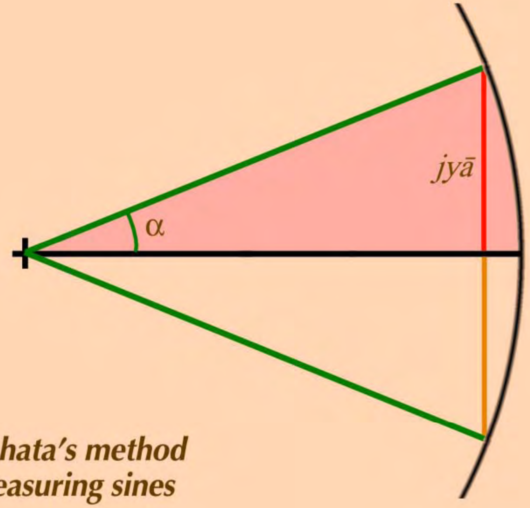
# Testimonies from two French mathematicians:

- “The ingenious method of expressing every possible number using a set of ten symbols (each symbol having a place value and an absolute value) emerged in India. The idea seems so simple nowadays that its significance and profound importance is no longer appreciated. Its simplicity lies in the way it facilitated calculation and placed arithmetic foremost amongst useful inventions. The importance of this invention is more readily appreciated when one considers that it was beyond the two greatest men of Antiquity, Archimedes and Apollonius.” — Laplace
- “The Indian mind has always had for calculations and the handling of numbers an extraordinary inclination, ease and power, such as no other civilization in history ever possessed to the same degree. So much so that Indian culture regarded the science of numbers as the noblest of its arts.... A thousand years ahead of Europeans, Indian savants knew that the zero and infinity were mutually inverse notions....” — Georges Ifrah

Âryabhata was a brilliant scientist who lived at Kusumapura (probably today’s Patna). He wrote the *Âryabhatîya,* a brief but extremely important treatise of mathematics and astronomy, at the age of 23! The date was 499 CE. A few highlights:

*makhi bhakhi phakhi dhakhi ṇakhi ñakhi*
*ṅakhi hasjha skaki kiṣga ghakhi kighva |*
*ghlaki kigra hakya dhaki kica*
*sga śjha ṅva kla pta pha cha kala-ardha-jyāḥ ||*

(*Âryabhatîya*, verse 12)




*Aryabhata’s method for measuring sines*

# Âryabhata also...

- **Gave a table of sines for every 3.75° (24 divisions per quadrant), in a special coded language where every syllable stands for a number *(left: all values up to 90°, correct to the 4th or 5th decimal)*.**
- **Proposed that $\pi$ = 62832 / 20000 = 3.1416, adding it was an “approximate” value.**
- **Gave for the first time the formula for the area of a triangle.**
- **Gave precise algorithms to extract square and cubic roots.**
- **Solved (in integers) *ax* + *c* = *by* (*kuttaka* or “pulverizing” method).**

# Brahmagupta (b. 598 CE)

"A thousand years ahead of Europeans, Indian savants knew that the zero and infinity were mutually inverse notions.…"

— Georges Ifrah

*Khachheda* means "divided by *kha*";
*Kha* (space) stands for zero;
"Divided by zero" = infinity.

— Brahmagupta,
*Brahmasphutasiddhânta* (628 CE)

Not a perfect definition of the mathematical infinite (it was formulated only in the 19th century), but the first great insight in the field.

- A study of right-angled triangles to provide geometric solutions to quadratic equations.
- Calculation of the diagonals of cyclic quadrilaterals.
- His *bhâvanâ* method worked out integral solutions for second-order indeterminate equations of the so-called Pell's equation ($17^{th}$ c.): $Nx^2 + 1 = y^2$ — known in India as *varga prakriti*. The *bhâvanâ* is an algorithm: once one set of solutions has been found, an infinite number of sets can be produced. (This infinity of solutions is wrongly attributed to the French mathematician Fermat.)
- Brahmagupta was translated into Arabic in Baghdad and spurred the growth of algebra in Arab countries and later in Europe too.

# Other foundations of modern algebra

- Mahâvira (9th century AD): approximate formulae for the area and circumference of an ellipse; work on permutations and combinations.
- Bhâskara II (12th century AD) developed the improved "cyclic method" (*chakravâla*); e.g., smallest solutions to $61x^2 + 1 = y^2$ are 226153980 & 1766319049. Lagrange reached the same solutions in the 18th century, but through a much longer method.
- Bhâskara II also worked on derivatives (of a sine function in relation to the velocity of planets).

# The Kerala School: advanced mathematics

- Infinite series, especially of trigonometric functions.
- Mâdhava (14th century): power series expansions for sine and cosine (correct to 1/3600th of a degree). Mâdhava knew the Gregory (1671) or Leibniz (1673) series for *arc tan x*, three centuries before them.
- Infinite series of $\pi$ (resulting in values with 10 correct decimals).
- Nîlakantha Somayâji (15th century): formula for the sum of a convergent infinite geometric series.

## The calculus controversy:

"Indian mathematics had on Arabic mathematics, and ultimately, through Latin translations, on European mathematics, an influence that is considerably neglected…. If indeed it is true that transmission of ideas and results between Europe and Kerala occurred [about calculus], then the 'role' of later Indian mathematics is even more important than previously thought.... The work of Indian mathematicians has been severely neglected by Western historians."

— British mathematician Ian Pearce

(www-history.mcs.st-andrews.ac.uk/history/Projects/Pearce/index.html)

# India's love affair with infinity: large numbers up to $10^{145}$ had specific Sanskrit names!

## India's love affair with the infinitesimal:

- Several texts define the *paramânu* unit (= "atom") as 262144 *paramânus* = 1 *angula*, which comes to 1 *paramânu* ≈ 70 nanometres!
- Smallest unit of weight in *Charaka Samhita*: 1*dvamshi* = 0.123 mg.
- Bhaskara II (in his *Siddhânta-shiromani* ) defines a unit of time (*truti*) equal to one 2,916,000,000th of a day or about 30 microseconds!
- What could have been the use of such tiny units of length, weight or time? A play of the mind? Some actual use?

**Some huge numbers conceived and named by Indian mathematicians**

| | | |
|---|---|---|
| 0 | zero : | *shūnya, bindu, kha, pūrna, vyoman...* |
| 1 | one : | *eka, ādi, pitāmaha, tanu, kshiti, indu...* |
| 2 | two : | *dvi, ashvin, netra, paksha...* |
| 3 | three : | *tri, guna, loka, kāla, agni...* |
| 4 | four : | *chatur, dish, yuga, iryā...* |
| 5 | five : | *pañcha, ishu, indriya, bhūta...* |
| 6 | six : | *shat, rasa, anga, shanmukha...* |
| 7 | seven : | *sapta, ashva, naga, rishi, sāgara, dvīpa...* |
| 8 | eight : | *ashta, gaja, nāga, mūrti...* |
| 9 | nine : | *nava, anka, graha, chhidra...* |
| 10 | ten : | *dasha, anguli, āshā, avatāra, dish...* |
| 100 | $10^{2}$ : | *shata* |
| 1000 | $10^{3}$ : | *sahasra* |
| 10000 | $10^{4}$ : | *ayuta, dashasahasra* |
| 100000 | $10^{5}$ : | *lakh, laksha, niyuta* |
| 1000000 | $10^{6}$ : | *dashalaksha, prayuta* |
| 10000000 | $10^{7}$ : | *koti, arbuda* |
| 100000000 | $10^{8}$ : | *arbuda, vyarbuda, nyarbuda, dashakoti* |
| 1000000000 | $10^{9}$ : | *padma, samudra, abja, ayuta, nahut* |
| 10000000000 | $10^{10}$ : | *kharva, madhya, arbuda, samudra* |
| 100000000000 | $10^{11}$ : | *nikharva, anta, madhya, ninnahut, salila* |
| 1000000000000 | $10^{12}$ : | *mahāpadma, parārdha, mahābjan antya* |
| 10000000000000 | $10^{13}$ : | *shanka, ananta, khamba, kankara* |
| 100000000000000 | $10^{14}$ : | *samudra, pakoti, jaladhi, padma, vādava* |
| 1000000000000000 | $10^{15}$ : | *madhya, akshiti, antya, mahāpadma* |
| 10000000000000000 | $10^{16}$ : | *antya, madhya, kshoni* |
| 100000000000000000 | $10^{17}$ : | *parārdha, abab, kshobhya, vrindhā* |
| 1000000000000000000 | $10^{18}$ : | *shanka* |
| 10000000000000000000 | $10^{19}$ : | *attata, vivahah* |
| 100000000000000000000 | $10^{20}$ : | *kshiti* |
| 1000000000000000000000 | $10^{21}$ : | *kotippakoti, kumud, utsanga* |
| 10000000000000000000000 | $10^{22}$ : | *kshoba* |
| 100000000000000000000000 | $10^{23}$ : | *bahula, gundhika* |
| 10000000000000000000000000 | $10^{25}$ : | *nāgabala, utpala* |
| 1000000000000000000000000000 | $10^{27}$ : | *pundarika, titilambha* |
| 10000000000000000000000000000 | $10^{28}$ : | *nahuta* |
| 100000000000000000000000000000 | $10^{29}$ : | *vyavasthānaprajñapati* |
| 10000000000000000000000000000000 | $10^{31}$ : | *hetuhila* |
| 1000000000000000000000000000000000 | $10^{33}$ : | *karahu* |
| 10000000000000000000000000000000000 | $10^{34}$ : | *mahāpadma* |
| 100000000000000000000000000000000000 | $10^{35}$ : | *hetvrindiya* |
| 10000000000000000000000000000000000000 | $10^{37}$ : | *samāptalambha* |
| 1000000000000000000000000000000000000000 | $10^{39}$ : | *gananāgati, kharva* |
| 100000000000000000000000000000000000000000 | $10^{41}$ : | *niravadya* |
| 1000000000000000000000000000000000000000000 | $10^{42}$ : | *akkhobhini* |
| 10000000000000000000000000000000000000000000 | $10^{43}$ : | *mudrābala* |
| 1000000000000000000000000000000000000000000000 | $10^{45}$ : | *sarvabala* |

and so on up to . . . $10^{145}$ !

# Advances in astronomy

## Âryabhata about the earth:

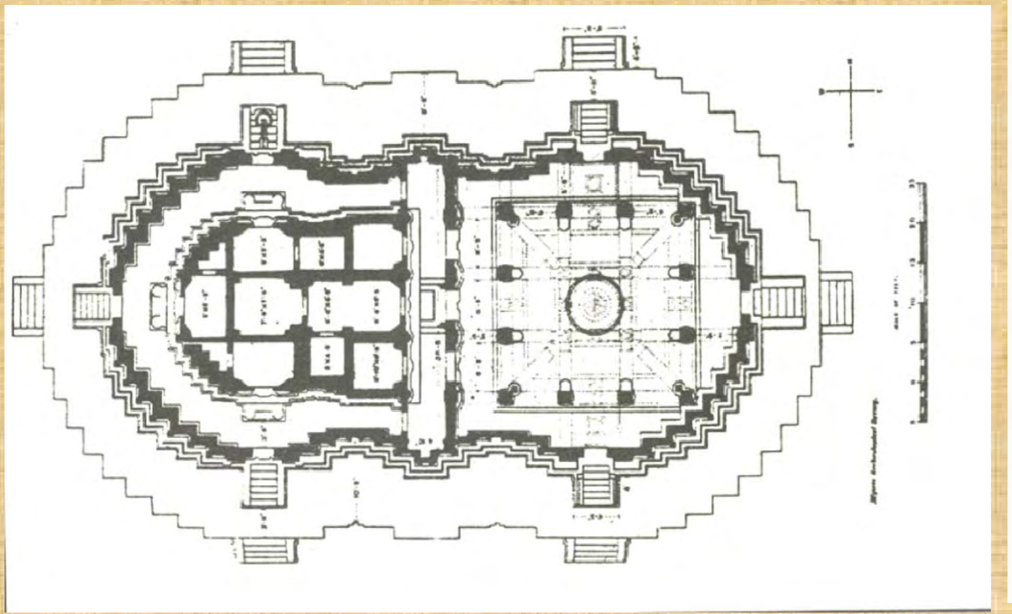
- The earth is a rotating sphere: the stars do not move, it is the earth that rotates. This causes the rising and setting of the sun. (Note that this is distinct from heliocentrism.)
- Its diameter is 1,050 *yojana* (1 *yojana* is 8,000 human heights or about 13.6 km). Its circumference is therefore 1,050 x 13.6 x $\pi$ = 44,860 km or 12% too large.
- His value for the moon's diameter is less precise: 4,284 km instead of 3,476 (23% error). His values for the Sun and the planets are far too small (he makes the five planets smaller than the moon).

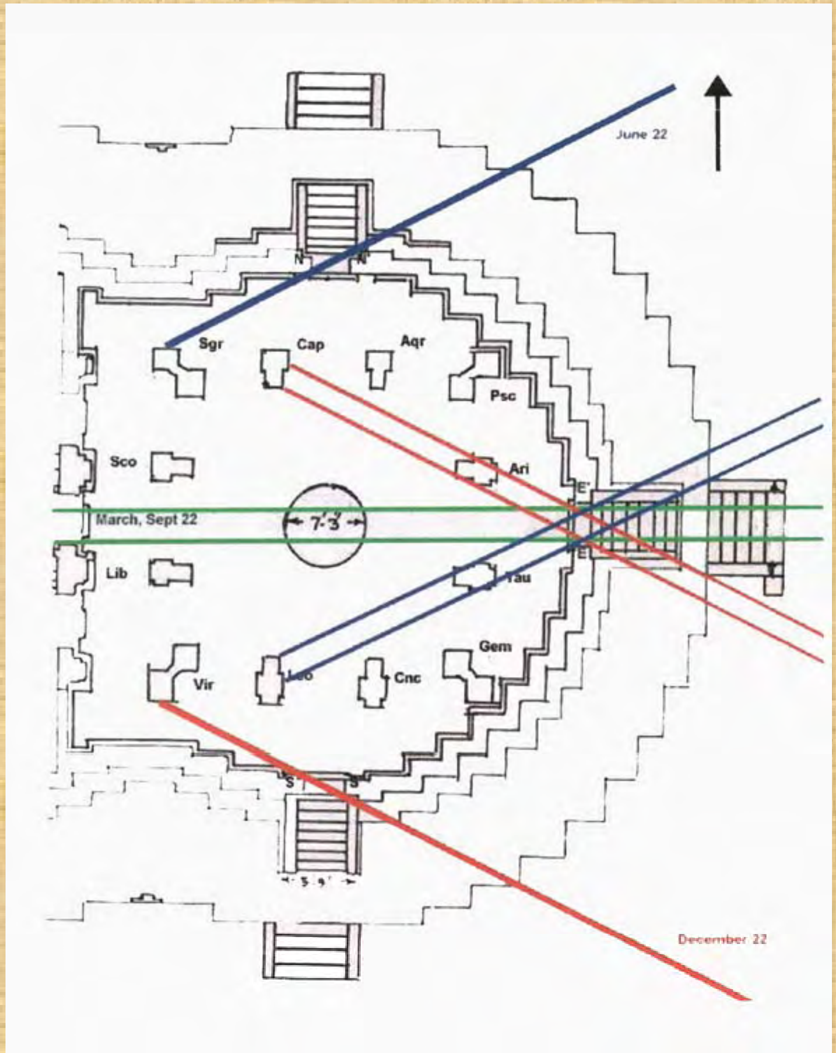
## Âryabhata on eclipses:

"The moon eclipses the sun, and the great shadow of the earth eclipses the moon."

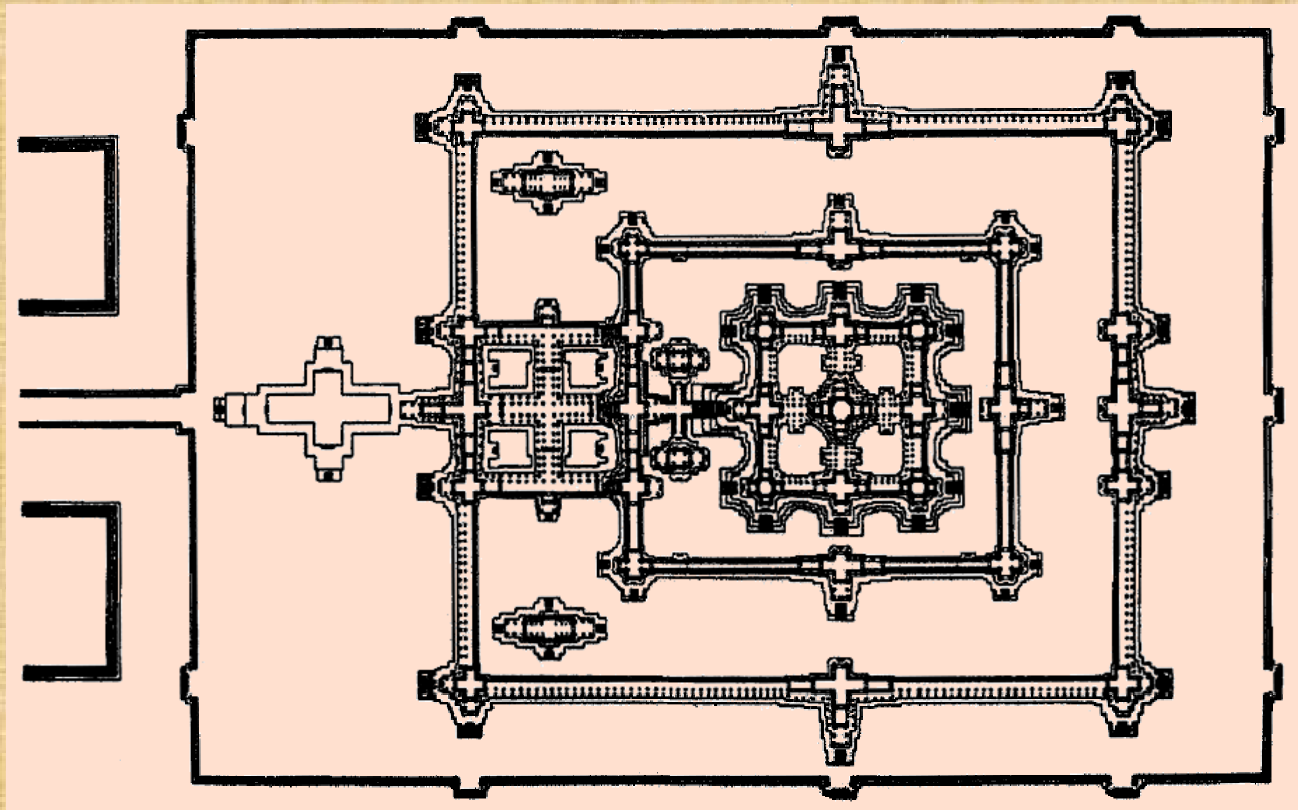
(*Âryabhatîya*, IV.37)

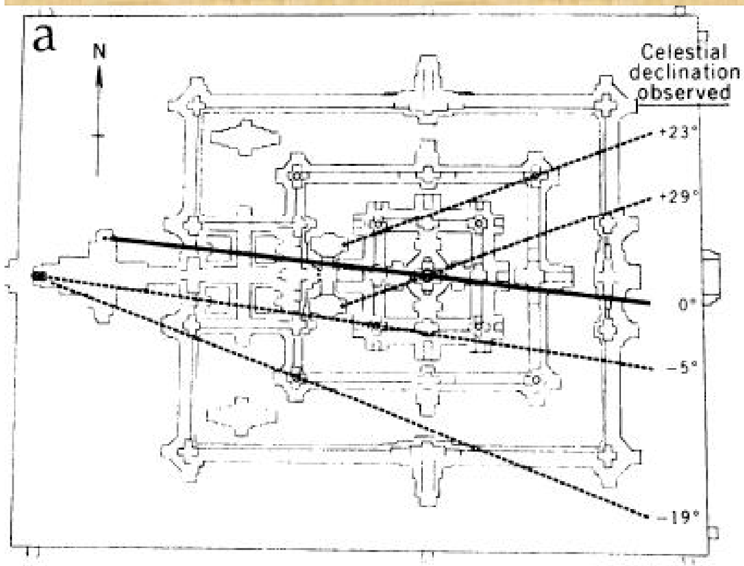
# Sringeri temple: astronomical alignments in temple architecture

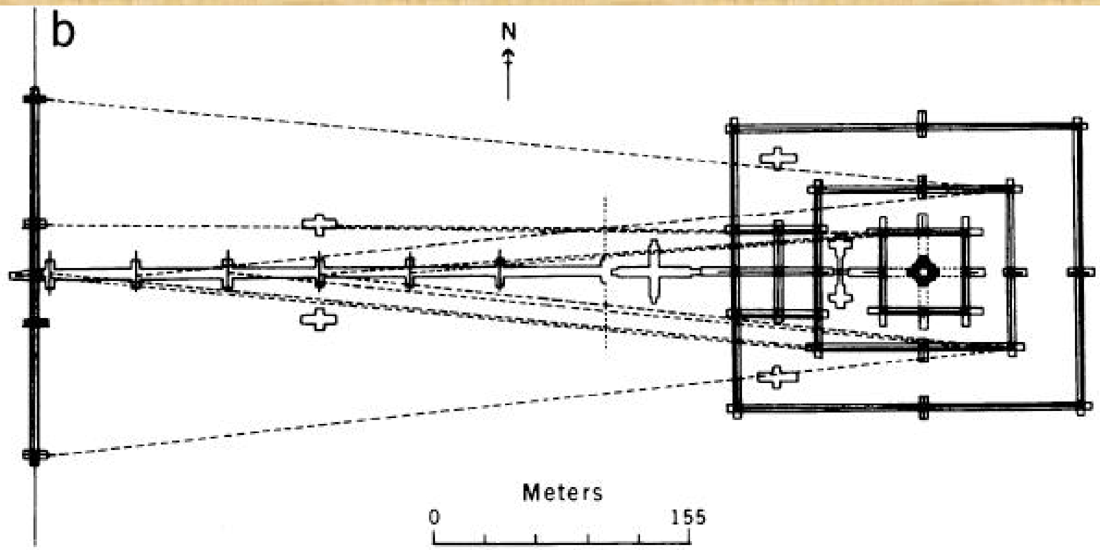




The ground plan of Angkor-Vat temple (the largest Hindu temple in the world, in Cambodia), with 22 alignments for observing the sun and the moon



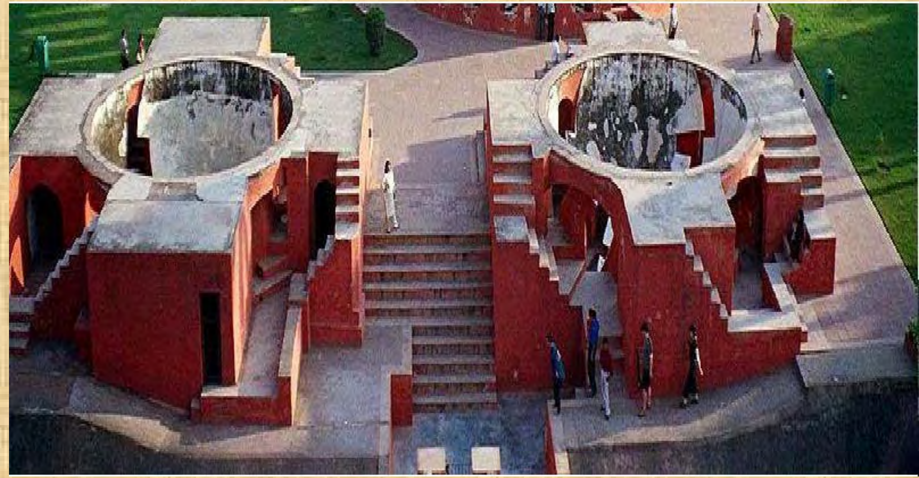


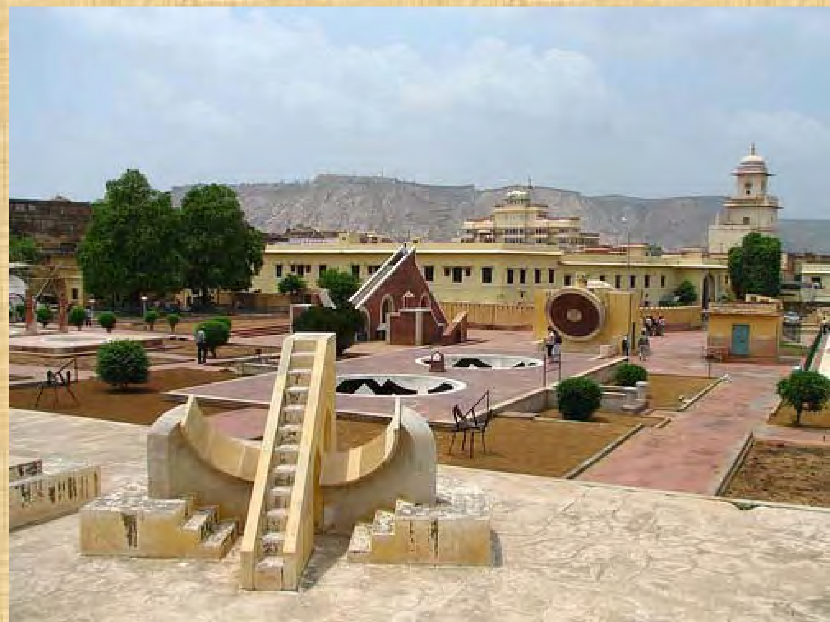



# The Kerala School: advanced astronomy

- Detailed calculations of orbits of sun, moon and planets: true and mean positions.
- Detailed study of eclipses (religious importance), including beginning, ending, middle point, duration, totality, etc.
- Excellent trigonometric approximations for equations of orbits (with corrections — *bîja samskâra* — introduced whenever necessary).
- Parameswara (1360-1465): observations of eclipses over 55 years and consequent correction techniques.
- Nîlakantha (c. 1500): planetary model of the sun orbiting the earth, with all other planets orbiting the sun: equivalent to heliocentrism. (This would also be Danish astronomer Tycho Brahe's system c. 1590.) Copernicus worked out his heliocentric system about 1510 and published it in 1543.

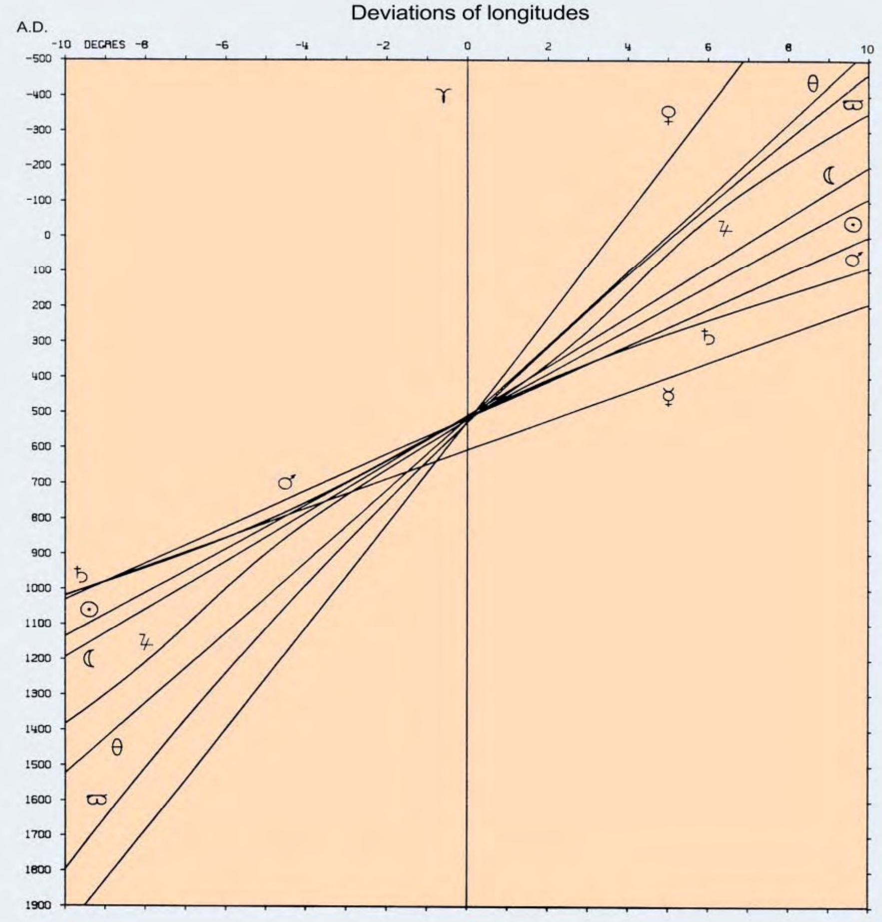
# Jai Singh's Jantar Mantar observatories (Delhi, Jaipur, Mathura, Ujjain and Varanasi, 18$^{th}$ century)



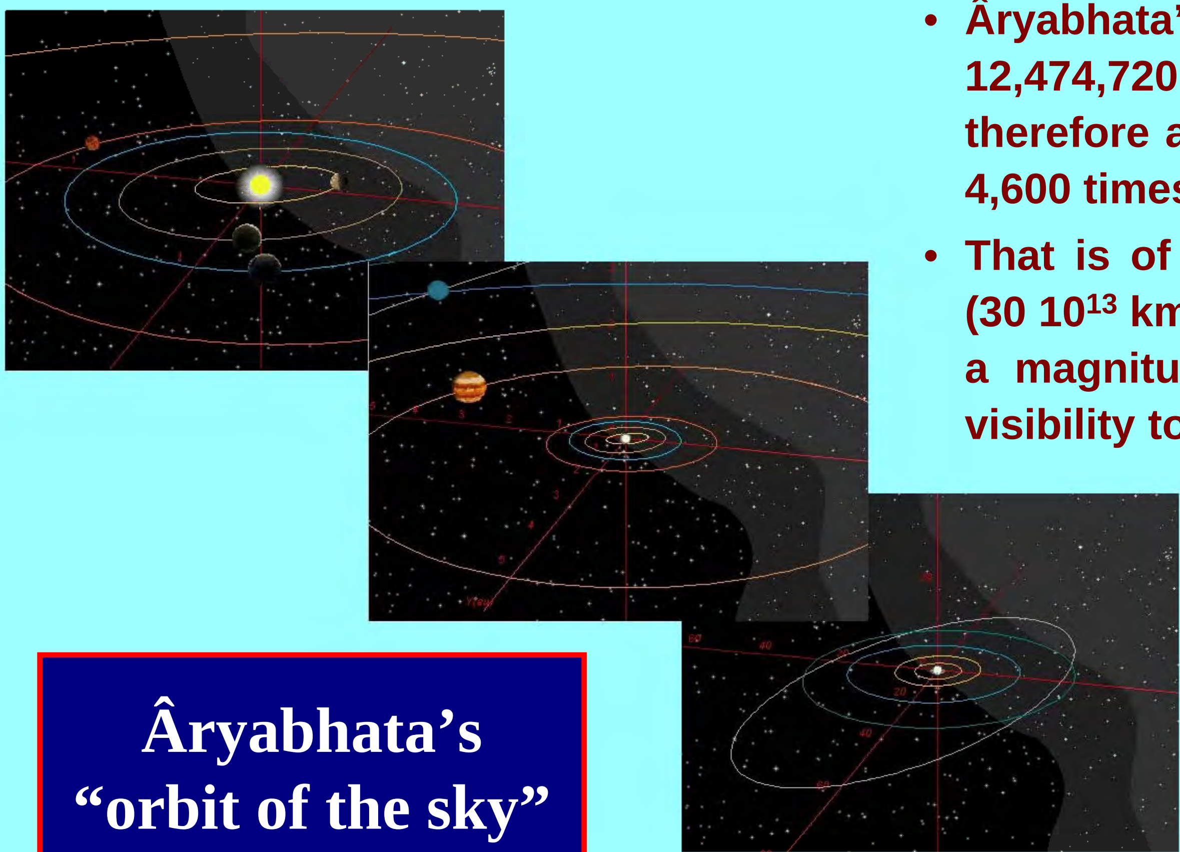


# Using modern science to investigate ancient traditions

**In 1971 Roger Billard (a French mathematician and Sanskritist) did a statistical study of the deviations of longitudes of Âryabhata's observations of planets. He proved that the deviations were smallest around 500 CE (except for Mercury, which is always hard to observe, being so close to the sun). This is precisely the date of the *Âryabhatîya*. This gave the lie to scholars who claimed that Âryabhata had borrowed his table of planetary positions from Babylonian astronomers.**
**A similar method is now being tested with more Indian astronomers.**




*Dating Aryabhata's observations through deviations of longitudes*
*(method of Roger Billard, 1971)*



# Âryabhata's "orbit of the sky"

- Âryabhata's "orbit of the sky" is 12,474,720,576,000 *yojanas* ≈ 16.8 $10^{13}$ km; therefore a diameter of 5.4 $10^{13}$ km, about 4,600 times our solar system's diameter.
- That is of the same order as 10 parsecs (30 $10^{13}$ km), a distance where the Sun has a magnitude of 4.7, almost the limit of visibility to the human eye.

Bhaskara I: "For us, the sky extends to as far as it is illumined by the rays of the Sun. Beyond that, the sky is immeasurable.... The sky is beyond limit; it is impossible to state its measure."

In other words, Âryabhata's "orbit of the sky" is of the same order of magnitude as the distance "illumined by the sun". An accident? If not, how could Âryabhata figure this out?

# More on the Indian scientific mind

- **We have seen the Indian notions of a cyclic, endless time, of limitless space, a fascination for large numbers: the concept of infinity underlies much of Indian science.**
- **On the other hand, it had little interest in axiomatics (unlike the Greeks), i.e., in a theoretical foundation of science. It was essentially empirical, i.e. relying on pragmatic methods. Some scholars have described this as a defect or limitation, others have pointed out that it allowed Indian science to progress faster towards precise results and innovative methods.**
- **Indian mathematicians had great skill for developing efficient algorithms. The word “algorithm” comes from the name of the Persian mathematician Al-Khwarizmi (780–850 CE), who wrote a book on Indian algebra largely based on Indian works.**
- **Nevertheless, a notion of proof *(upapatti)* does exist in the Kerala School, especially in the commentaries (e.g. Jyesthadeva’s *Yuktibhâsâ* of 1530 CE): observed results must be validated.**
- **The great mathematician S. Ramanujan produced hundreds of “theorems” without proof; he said they were intuitions from the family goddess Namagiri. Had he spent time on proofs, he might not be as creative.**

# Conclusion:
# The growth of a truly scientific spirit

- Bhâskara II: “It is necessary to speak out the truth accurately before those who have implicit faith in tradition. It will be impossible to believe in whatever is said earlier unless every erroneous statement is criticized and condemned.”
- In Europe at the same time, science was strangled by religious dogmas (remember how Galileo was compelled to condemn the heliocentric Copernican system which he knew to be true: otherwise he would have been burned at the stake like Giordano Bruno).
- India’s scientific advances were relayed to Europe by the Arabs (who translated many Indian texts) and contributed much to the birth of modern science. This contribution has not yet been fully assessed.
- Yet in India, the study of ancient science remains neglected.

# A few suggested readings
(for those who wish to explore further)


## General

- *A Concise History of Science in India*, ed. D. M. Bose, S. N. Sen & B. V. Subbarayappa (Indian National Science Academy, 1989)
- *Indian Science and Technology in the Eighteenth Century,* Dharampal (Other India Bookstore)
- *Encyclopaedia of Classical Indian Sciences*, ed. Helaine Selin & Roddam Narasimha (Universities Press, 2007)

## Mathematics

- *Indian Mathematics and Astronomy – Some Landmarks*, S. Balachandra Rao (Jnana Deep Publications, 3rd ed. 2004)
- *The Universal History of Numbers: From Prehistory to the Invention of the Computer,* George Ifrah (Penguin Books, 2005, 3 vols.)
- *The Crest of the Peacock,* George Gheverghese Joseph (Penguin Books, 2000)
- *Geometry in Ancient and Medieval India*, T. A. Sarasvati Amma (Motilal Banarsidass, 1999)
- *Lilavati of Bhaskaracarya,* translators K. S. Patwardhan, S. A. Naimpally & S. L. Singh (Motilal Banarsidass, 2001)
- *Computing Science in Ancient India*, eds. T. R. N. Rao & Subhash Kak (Munshiram Manoharlal Publishers 2000)

## Chemistry

- *A History of Hindu Chemistry*, Acharya Praffullachandra Ray (Shaibya Prakashan Bibhag, centenary edition 2002)
- *Chemistry and Chemical Techniques in India*, ed. B.V. Subbarayappa (Centre for Studies in Civilizations, vol. IV, part 1, 1999)

## Astronomy


- *The Sûrya Siddhânta: a Textbook of Hindu Astronomy*, tr. & ed. by Ebenezer Burgess & Phanindralal Gangooly (Motilal Banarsidass, 2000)
- *History of Astronomy in India*, ed. S. N. Sen & K. S. Shukla (Indian National Science Academy, 1985)
- *Indian Astronomy – An Introduction*, S. Balachandra Rao (Universities Press, 2000)
- *Indian Astronomy – A Primer*, S. Balachandra Rao (Bhavan's Gandhi Centre of Science and Human Values, 2008)
- *Bhaskara I and His Astronomy*, S. Balachandra Rao (Rashtriya Sanskrit Vidyapeetha, 2003)
- *Aryabhata I and His Astronomy*, S. Balachandra Rao (Rashtriya Sanskrit Vidyapeetha, 2003)
- *Ancient Cities, Sacred Skies: Cosmic Geometries and City Planning in Ancient India*, J. McKim Malville & Lalit M. Gujral (IGNCA & Aryan Books International, 2000)
- *Jai Singh and His Astronomy,* Virendra Nath Sharma (Motilal Banarsidass, 1995)


## Technology


- *History of Technology in India*, ed. A. K. Bag (Indian National Science Academy, 1997)
- *Delhi Iron Pillar: New Insights*, R. Balasubramaniam (Indian Institute of Advance Study & Aryan Books International, 2002)
- *Marvels of Indian Iron through the Ages*, R. Balasubramaniam (Rupa & Infinity Foundation, 2008)
- *The Rustless Wonder: A Study of the Iron Pillar at Delhi,* T. R. Anantharaman (Vigyan Prasar 1996)
- *India's Legendary Wootz Steel: An Advanced Material of the Ancient World*, Sharada Srinivasan & Srinivasa Ranganatha (NIAS & IISc, 2004, see http://met.iisc.ernet.in/~rangu/ )
- *History of Iron Technology in India: From Beginning To Pre-Modern Times*, Vibha Tripathi (Rupa & Infinity Foundation, 2008)

# A few Internet resources

- www-history.mcs.st-andrews.ac.uk/history/Indexes/Indians.html
  (and all links on that page)
- http://en.wikipedia.org/wiki/Indian_mathematics
- http://en.wikipedia.org/wiki/List_of_Indian_mathematicians
- http://en.wikipedia.org/wiki/Indian_astronomy
- www.insa.ac.in/html/home.asp
- www-history.mcs.st-andrews.ac.uk/history/Projects/Pearce/index.html
- www.infinityfoundation.com/mandala/tks_essays_frameset.htm
- www.indianscience.org
  (and in particular: www.indianscience.org/essays/essays.shtml)